13A

1 (a) $K(-2,5)$ $L(4,7)$
$\quad x_1 \; y_1 \qquad x_2 \; y_2$

$$M_{KL} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{7-5}{4-(-2)}$$

$$= \frac{2}{6}$$

$$M_{KL} = \frac{1}{3}$$

(b) $M(-7,-2)$, $N(-1,0)$
$\quad x_1 \; y_1 \qquad x_2 \; y_2$

$$M_{MN} = \frac{0-(-2)}{-1-(-7)}$$

$$= \frac{2}{6}$$

$$M_{MN} = \frac{1}{3}$$

(c) equal gradient $\therefore$ parallel.

2(a) $2x + 3y = 7$

$$3y = -2x + 7$$

$$y = -\tfrac{2}{3}x + \tfrac{7}{3}$$

$$M_{AB} = -\frac{2}{3}$$

13A

2(b) $c = 7/3$

$(0, \frac{7}{3})$

2(c) $M_{CD} = -\frac{2}{3}$ $(9,-3)$

$y - b = m(x-a)$

$y - -3 = -\frac{2}{3}(x - 9)$

$y+3 = -\frac{2}{3}(x-9)$

$3(y+3) = -2(x-9)$

$3y+9 = -2x+18$

$3y+2x-9 = 0$

or

$y+3 = -\frac{2}{3}x + 6$

$y = -\frac{2}{3}x + 3$

(3) $3x + 2y = 5$

$2y = -3x + 5$

$y = \frac{-3}{2}x + \frac{5}{2}$

$M_{EF} = \frac{-3}{2}$

$y-b = m(x-a)$ $(-2,2)$

$y - 2 = \frac{-3}{2}(x - -2)$

$2(y-2) = -3(x+2)$

$2y-4 = -3x-6$ $\Rightarrow$ $2y + 3x + 2 = 0$

or

$y-2 = -\frac{3}{2}x - 3$

$y = -\frac{3}{2}x - 1$

13A

(6) $ax - 2y + 7 = 0$

$ax + 7 = 2y$

$2y = ax + 7$

$y = \frac{a}{2}x + \frac{7}{2}$

$m = \frac{a}{2}$

$3x + 4y = 9$

$4y = -3x + 9$

$y = -\frac{3}{4}x + \frac{9}{4}$

$m = -3/4$

as parallel

$\frac{a}{2} = -\frac{3}{4}$

$a = -\frac{6}{4}$

$a = \frac{-3}{2}$

(7) $M_{AD} = \frac{-2-(-3)}{a-(-1)}$ $\qquad$ $M_{BC} = \frac{6-5}{7-2}$

$= \frac{1}{a+1}$ $\qquad$ $= \frac{1}{5}$

$M_{AD} = M_{BC}$

$\frac{1}{a+1} = \frac{1}{5} \Rightarrow 5 = a+1 \Rightarrow a = 4$

13B

1 (a) $m_{AB} = \frac{3-0}{0--3}$

$= \frac{3}{3}$

$= 1$

$m_{BC} = \frac{7-3}{3-0}$

$= \frac{4}{3}$

$m_{AB} \neq m_{BC}$ $\therefore$ A, B & C are not collinear

1 (c) $m_{GH} = \frac{-3-0}{-2-4}$

$= \frac{-3}{-6}$

$= \frac{1}{2}$

$m_{HJ} = \frac{-4-(-3)}{-4-(-2)}$

$= \frac{-1}{-2}$

$= \frac{1}{2}$

$m_{GH} = m_{HJ}$ $\therefore$ GH // HJ and since H is a common point G, H and J are collinear

2 $m_{PQ} = \frac{1-(-3)}{-1-(-5)}$

$= \frac{4}{+6}$

$= \frac{2}{3}$

$m_{QR} = \frac{k-1}{10-1}$

$= \frac{k-1}{9}$

$\frac{k-1}{9} = -2$, $k-1 = -18$, $k = -17$

13B

(2) $m_{PQ} = \frac{1-(-3)}{1-(-5)}$

$= \frac{4}{6}$

$= \frac{2}{3}$

$m_{QR} = \frac{k-1}{10-1}$

$= \frac{k-1}{9}$

collinear so $m_{PQ} = m_{QR}$

$\frac{k-1}{9} = \frac{2}{3}$

$k-1 = \frac{2}{3} \times 9$

$k-1 = 6$

$k = 7$

(3) $A(5,-5)$ $B(-1,10)$

$m_{AB} = \frac{10-(-5)}{-1-5}$

$= \frac{15}{-6}$

$= \frac{-5}{2}$

13B

(3) continued

$C(3, -1)$

$$m_{AC} = \frac{-1-(-5)}{5-3}$$

$$= \frac{4}{2}$$

$$= 2$$

$m_{AC} \neq m_{AB}$ $\therefore$ A, B, C not collinear. The fly did not walk over $(3, -1)$

$D(3, 0)$

$$m_{AD} = \frac{0-(-5)}{5-3}$$

$$= \frac{5}{2}$$

$m_{AB} = m_{AD}$ $\therefore$ $AB \parallel AD$ and since A is a common point A, B and D are collinear. The fly did walk over $(3, 0)$.

13B

(4) $m_{AB} = \frac{-2-13}{-3-6}$

$= \frac{-15}{-9}$

$= \frac{5}{3}$

$m_{BS} = \frac{-2-(-7)}{-3-(-6)}$

$= \frac{5}{3}$

$m_{AB} = m_{BS}$ $\therefore$ AB is parallel to BS and since B is a common point A, B and S are collinear. Team 1 will make it to the station.

$m_{CD} = \frac{-11-(-17)}{0-(-9)}$

$= \frac{6}{9}$

$= \frac{2}{3}$

$m_{CS} = \frac{-7-(-17)}{-6-(-9)}$

$= \frac{10}{3}$

$m_{CD} \neq m_{CS}$ $\therefore$ C, D and S are not collinear. Team 2 will not make it to the station.

13C

1(a) $m = \frac{2}{3}$

$m_{perp} = -\frac{3}{2}$ (flip fraction and change sign)

(d) $m = 7$

$m_{perp} = -\frac{1}{7}$

(f) $m = -3$

$m_{perp} = \frac{1}{3}$

(h) $m = 0$ (horizontal)

$m_{perp} = \frac{1}{0}$ = undefined (vertical)

2 $m_{EG} = \frac{12-3}{-2(-4)}$

$= \frac{9}{2}$

$m_{perp} = \frac{-2}{9}$

13G

(4) $m_{ST} = \dfrac{-5-1}{-6-2}$

$= \dfrac{-6}{-8}$

$= \dfrac{3}{4}$

$m_{perp} = \dfrac{-4}{3}$

midpoint ST :

S		mid		T
2	-4	-2	-4	-6
1	-3	-2	-3	-5

$M(-2,-2)$

$y-b = m(x-a)$

$y - -2 = \dfrac{-4}{3}(x - -2)$

$y+2 = \dfrac{-4}{3}(x+2)$

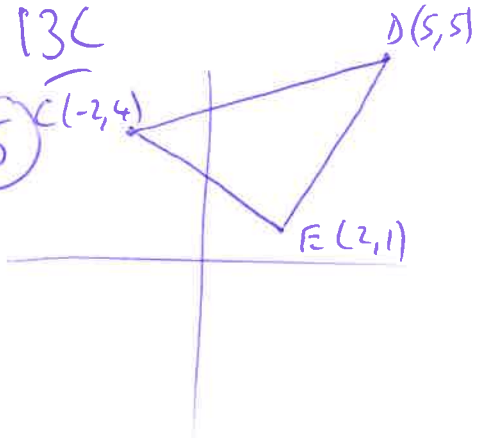
$3(y+2) = -4(x+2)$

$3y+6 = -4x-8$

$3y+4x = -14$

13C

5



$\angle CED$ looks right angled.

$$m_{CE} = \frac{1-4}{2--2}$$

$$= \frac{-3}{4}$$

$$m_{DE} = \frac{1-5}{2-5}$$

$$= \frac{-4}{-3}$$

$$= \frac{4}{3}$$

$$m_{CE} \times m_{DE} = \frac{-3}{4} \times \frac{4}{3}$$

$$= -1$$

$\therefore$ CE is perpendicular to DE and ΔCDE is right angled at E.

13C

(7) $5y - 2x = 4$

$5y = 2x + 4$

$y = \frac{2}{5}x + \frac{4}{5}$

$m = \frac{2}{5}$

$2y + 5x = 4$

$2y = -5x + 4$

$y = -\frac{5}{2}x + 2$

$m = -\frac{5}{2}$

$m_1 \times m_2 = \frac{2}{5} \times -\frac{5}{2}$

$= -1$

$\therefore$ The lines are perpendicular

13C

(8) $7y - ax = 13$

$7y = ax + 13$

$y = \frac{a}{7}x + \frac{13}{7}$

$m = \frac{a}{7}$

$5y - 7x = 24$

$5y = 7x + 24$

$y = \frac{7}{5}x + \frac{24}{5}$

$m = \frac{7}{5}$

lines perpendicular so $\frac{a}{7} \times \frac{7}{5} = -1$

$\frac{a}{5} = -1$

$\underline{\underline{a = -5}}$

13C

10 (a) $l_1: \quad 2y = x + 6$

$$y = \frac{1}{2}x + 3$$

$$m_{l_1} = \frac{1}{2}$$

l_1 parallel to l_2

$$m_{l_2} = \frac{1}{2} \qquad (0, -2)$$

$$y - b = m(x - a)$$

$$y - (-2) = \frac{1}{2}(x - 0)$$

$$y + 2 = \frac{1}{2}x$$

$$2(y+2) = x$$

~~2+y+~~

$$2y + 4 = x$$

$$\underline{2y = x - 4}$$

(b) At A $y=0$ (as on x-axis)

$$x - 4 = 0$$

$$x = 4$$

$$\underline{A(4, 0)}$$

13C

10(b) $m_{AB} = -2$ (perpendicular to $m_{l_1} = \frac{1}{2}$)

$A(4,0)$

$y - b = m(x-a)$

$y - 0 = -2(x-4)$

$y = -2x + 8$

At B l_1 intersects AB, find B using sim equations.

$y = -2x + 8$ ①

$2y = x + 6$ ②

$y + 2x = 8$ ① (rearranged)

$2y - x = 6$ ② (rearranged)

$2y + 4x = 16$ ③ (① × 2)

$5x = 10$ ③ − ②

$x = 2$

sub in ①

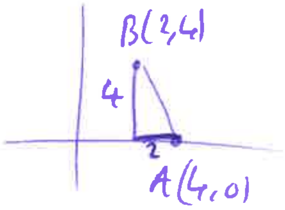
$y + 2(2) = 8$

$y = 4$

$B(2,4)$

13C

10(c) Shortest distance given by length of AB as AB is perpendicular to l_1 and l_2



$$|AB|^2 = 2^2 + 4^2$$

$$AB^2 = 20$$

$$AB = \sqrt{20}$$

shortest distance $= \sqrt{20}$

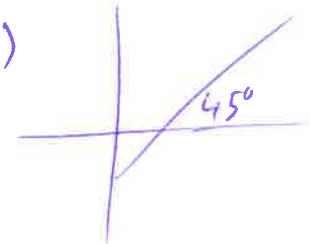
$$= \sqrt{4}\sqrt{5}$$

$$= 2\sqrt{5}$$

(always simplify a surd where possible)

13D

1(a)

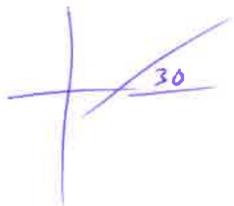


$m = \tan\theta$

$m = \tan 45$

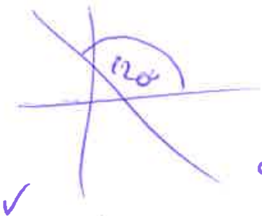
$\underline{m = 1}$

1(b) $m = \tan 30$



$\underline{m = \frac{1}{\sqrt{3}}}$

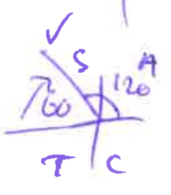
(d) $m = \tan 120$



← negative gradient

$m = -\tan 60$

$\underline{m = -\sqrt{3}}$



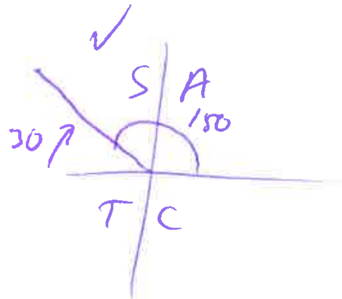
1(e) $m = \tan 90$

$m = \underline{\text{undefined}}$

(vertical line)

13D

1(f) $m = \tan 150$

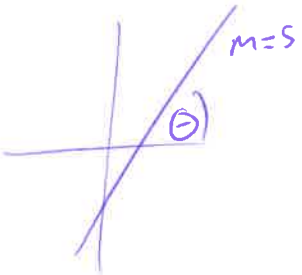


negative gradient

$m = -\tan 30$

$m = -\dfrac{1}{\sqrt{3}}$

2(a)



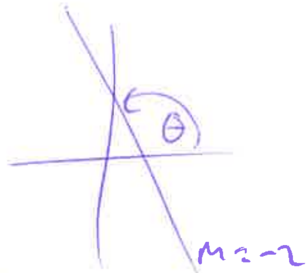
$m = \tan\theta$

$5 = \tan\theta$

$\theta = \tan^{-1}(5)$

$\theta = 78.7°$

(c) $m = \tan\theta$



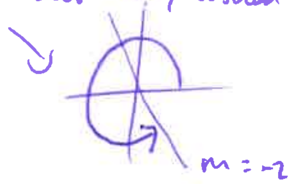
$-2 = \tan\theta$

$\tan^{-1}(2) = 63.4°$

$\theta = 180 - 63.4°$

$\theta = 116.6°$

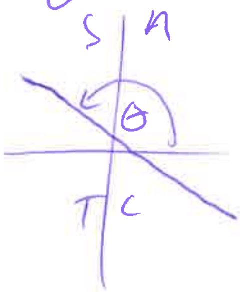
Note: solution in 4^{th} quadrant would be $360 - 63.4 = 296.6°$ but only asked for $\theta = 116.6°$



13D

2(d) $m = \tan\theta$

$-\frac{2}{5} = \tan\theta$



$\tan^{-1}\left(\frac{2}{5}\right) = 21.8°$

$\theta = 180 - 21.8$

$= \underline{\underline{158.2°}}$

3. $m_{AB} = \frac{4-1}{4-(-5)}$

$= \frac{3}{9}$

$= \frac{1}{3}$

$m = \tan\theta$

$\frac{1}{3} = \tan\theta$

$\theta = \tan^{-1}\left(\frac{1}{3}\right)$

$\theta = \underline{\underline{18.4°}}$

13D

(5) $M_{OA} = \frac{7-0}{3-0}$

$= \frac{7}{3}$

$\frac{7}{3} = \tan\theta_{OA}$

$\theta_{OA} = \tan^{-1}\left(\frac{7}{3}\right)$

$\theta_{OA} = 66.80^\circ$

$M_{OB} = \frac{2-0}{5-0}$

$= \frac{2}{5}$

$\frac{2}{5} = \tan\theta_{OB}$

$\theta_{OB} = \tan^{-1}\left(\frac{2}{5}\right)$

$\theta_{OB} = 21.80^\circ$

$\angle AOB = 66.80^\circ - 21.80^\circ$

$= \cancel{47.0^\circ}$

$= 45^\circ$

13D

6

$$m_{JK} = \frac{9-(-9)}{4--2}$$

$$= \frac{18}{6}$$

$$= 3$$

$$m = \tan\theta$$

$$3 = \tan\theta_{JK}$$

$$\theta_{JK} = \tan^{-1}(3)$$

$$\underline{\theta_{JK} = 71.57^\circ}$$

$$m_{PQ} = \frac{5-(-3)}{9-(-7)}$$

$$= \frac{8}{16}$$

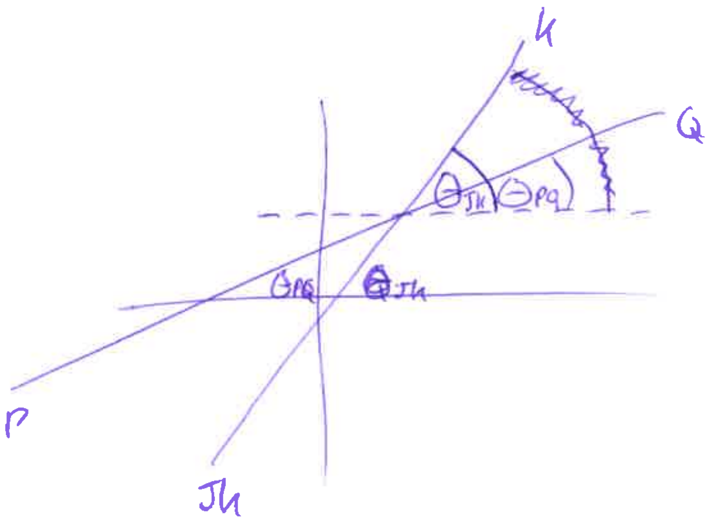
$$= \frac{1}{2}$$

$$m_{PQ} = \tan\theta$$

$$\frac{1}{2} = \tan\theta_{PQ}$$

$$\theta_{PQ} = \tan^{-1}\left(\frac{1}{2}\right)$$

$$\underline{\theta_{PQ} = 26.57^\circ}$$



angle between $= \theta_{JK} - \theta_{PQ}$

$= 71.57^\circ - 26.57^\circ$

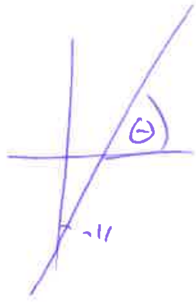
$= \underline{\underline{45^\circ}}$

13D

(7) $y - 7x + 11 = 0$

$$y = 7x - 11$$

$$m = 7$$



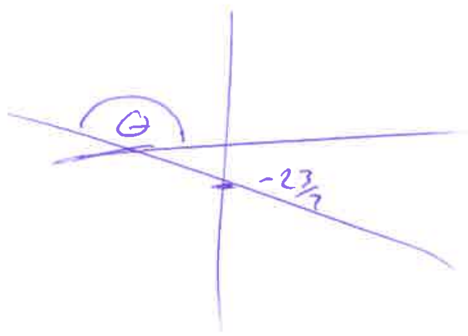
$$m = \tan\theta$$
$$7 = \tan\theta$$
$$\theta = \tan^{-1}(7)$$
$$\theta = 81.87^\circ$$

$$7y + x = 23$$

$$7y = -x - 23$$

$$y = -\frac{1}{7}x - \frac{23}{7}$$

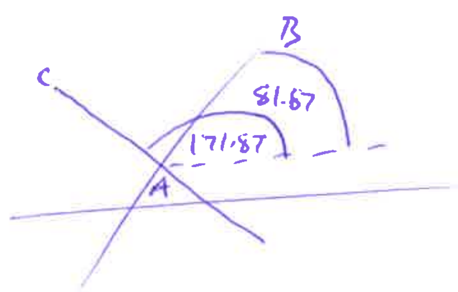
$$m = -\frac{1}{7}$$



$$-\frac{1}{7} = \tan\theta$$

$$\cancel{\theta} \quad \tan^{-1}\left(\frac{1}{7}\right) = 8.13^\circ$$

$$\theta = 180 - 8.13^\circ$$
$$= 171.87^\circ$$



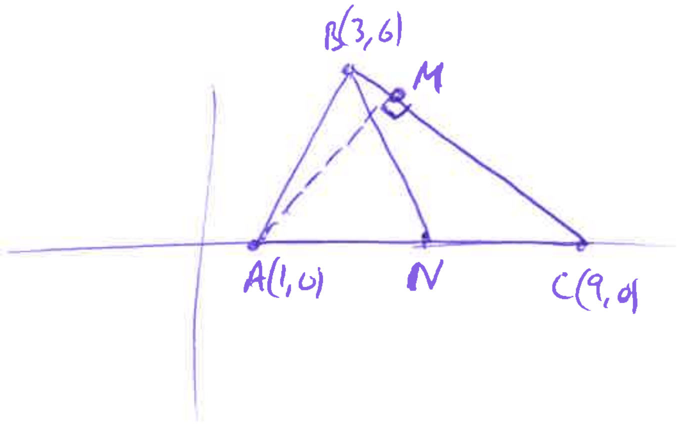
$$\angle BAC = 171.87^\circ - 81.87^\circ$$
$$= \underline{\underline{90^\circ}}$$

Note: $m_1 = 7 \quad m_2 = -\frac{1}{7}$

$m_1 \times m_2 = 7 \times -\frac{1}{7} = -1 \quad \therefore$ AB perp to BC so angle $= 90^\circ$ is valid alternative solution.

13F ①

Always make a sketch



(a) N = mid AC

A	N	C
1	5	9
0	0	0

$N(5,0)$

$$m_{BN} = \frac{6-0}{3-5}$$

$$= \frac{6}{-2}$$

$$= -3$$

$y - b = m(x-a)$ (can use B or N as both lie on BN).

$N(5,0)$

$$y - 0 = -3(x-5)$$

$$\underline{\underline{y = -3x + 15}}$$

13F

1(b) look back to diagram on previous page.

m_{AM} is perpendicular to m_{BC}

$$m_{BC} = \frac{0-6}{9-3}$$
$$= \frac{-6}{6}$$
$$= -1$$

$$m_{AM} = 1$$

$y - b = m(x-a)$ A and M lie on AM, we do not know M so must use A(1,0)

$$y - 0 = 1(x-1)$$
$$\underline{\underline{y = x - 1}}$$

1(c) BN: $y = -3x + 15 \Rightarrow y + 3x = 15$ (1)

AM: $y = x - 1$ $\qquad y - x = -1$ (2)

$4x = 16$ (1) − (2)

$x = 4$

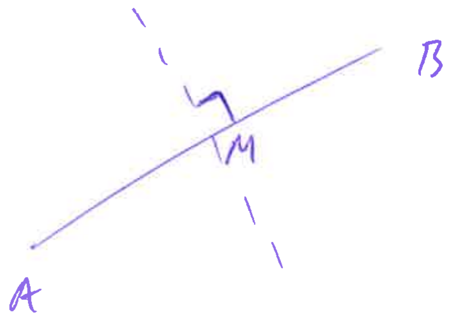
sub in (2) $y - 4 = -1$

$y = 3$

$$\underline{\underline{P(4,3)}}$$

13F

2



M :

A	+2	M	+2	B
−1		1		3
1	+3	4	+3	7

$M(1,4)$

$$m_{AB} = \frac{7-1}{3-(-1)}$$

$$= \frac{6}{4}$$

$$= \frac{3}{2}$$

$$m_{perp} = -\frac{2}{3}$$

$$y - b = m(x - a)$$

must use $M(1,4)$ as it is only point we know on the perpendicular bisector.

$$y - 4 = -\frac{2}{3}(x-1)$$

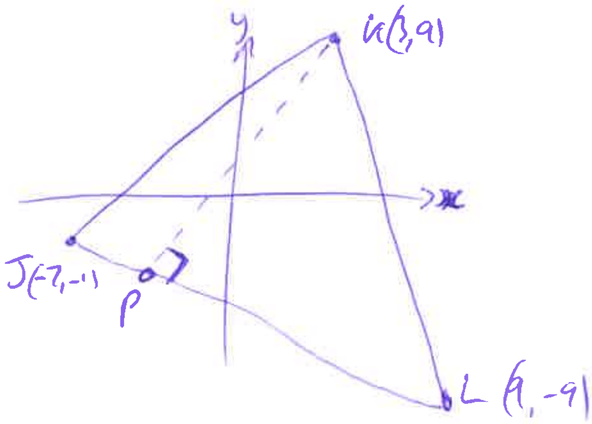
$$3(y-4) = -2(x-1)$$

$$3y - 12 = -2x + 2$$

$$3y + 2x = 14$$

(or $\frac{3}{2}y + x = 7$ but integer coefficients are preferred).

13F
3



(a) JL: $m_{JL} = \frac{-9-(-1)}{9-(-7)}$

$= \frac{-8}{16}$

$= -\frac{1}{2}$

$y - b = m(x-a)$

$y-(-1) = -\frac{1}{2}(x - -7)$

$y+1 = -\frac{1}{2}(x+7)$

$2(y+1) = -(x+7)$

$2y+2 = -x-7$

$2y = -x-9$

$\underline{\underline{2y + x = -9}}$

(b) KP: $m_{JL} = -\frac{1}{2}$

$m_{KP} = 2$ as m_{JL} is perpendicular KP.

K(3,9) lies on line KP

$y - b = m(x-a)$

13F

3(b) continued

$$y - 9 = 2(x - 3)$$
$$y - 9 = 2x - 6$$
$$y = 2x + 3$$
$$\underline{y - 2x = 3}$$

3(c) at P KP and JL intersect.

KP: $y - 2x = 3$ ①

JL: $2y + x = -9$ ②

$2y - 4x = 6$ ③ (① × 2)

$5x = -15$ ② − ③

$\underline{x = -3}$

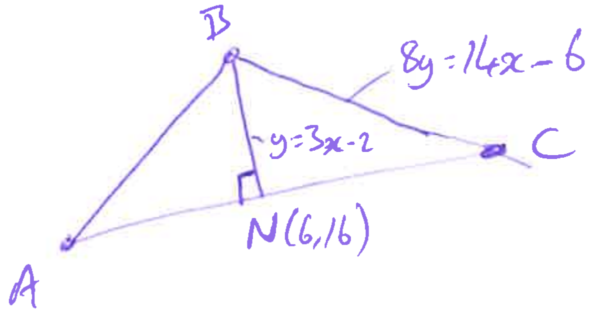
sub $x = -3$ in ①

$$y - 2(-3) = 3$$
$$y + 6 = 3$$
$$\underline{y = -3}$$

$$\underline{P(-3, -3)}$$

13F

5



Note: Always make a sketch when unsure. Use sketch to develop a relevant strategy.

BC : $8y = 14x - 6$

BN : $y = 3x - 2$

Find C. Find equation of NC (or AC), find point of intersection of BC and NC.

NC : $m_{BN} = 3$

$m_{NC} = -\frac{1}{3}$ as BN perpendicular to NC

use N as N lies on NC

$$y - 16 = -\frac{1}{3}(x - 6)$$

$$3y - 48 = -(x - 6)$$

$$3y - 48 = -x + 6$$

NC: $3y + x = 54$ ①

BC : $8y - 14x = -6$ ②

$42y + 14x = 756$ ③ (① × 14)

$50y = 750$ ② + ③

$y = 15$

13F

5 continued

sub $y = 15$ into 1

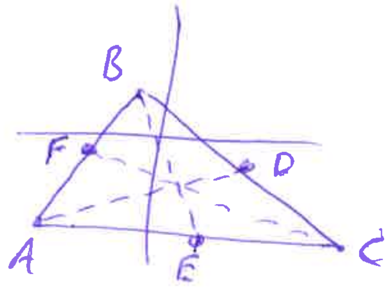
$3(15) + x = 54$

$45 + x = 54$

$x = 9$

$C(9, 15)$

13F (6)



AD

Mid BC

B	D	C
-1	3	7
2	-1	-4

D(3, -1)

$$m_{AD} = \frac{-4-(-1)}{-3-3}$$
$$= \frac{-3}{-6}$$
$$= \frac{1}{2}$$

$y - b = m(x-a)$ use D(3, -1) (or A(-3, -4))

$y - -1 = \frac{1}{2}(x-3)$

$y + 1 = \frac{1}{2}(x-3)$

$2(y+1) = x-3$

$2y + 2 = x - 3$

$2y - x = -5$

BE mid AC

A	E	C
-3	2	7
-4	-4	-4

E(2, -4)

13F

6 continued (ii)

$$m_{BE} = \frac{-4-2}{2--1}$$

$$= \frac{-6}{3}$$

$$= -2$$

$y - b = m(x - a)$ use $B(-1, 2)$ or $E(2, -4)$

$$y - 2 = -2(x - -1)$$

$$y - 2 = -2(x+1)$$

$$y - 2 = -2x - 2$$

$$\underline{\underline{y = -2x}}$$

~~CE~~ CF mid AB

A	F	B
-3	-2	-1
-4	-1	2

$F(-2, -1)$

$$m_{CF} = \frac{-1 - -4}{-2 - 7}$$

$$= \frac{3}{-9}$$

$$= -\frac{1}{3}$$

13F

6 continued (iii)

$y - b = m(x - a)$ using $C(7,-4)$ (or $F(-2,-1)$)

$y - -4 = -\frac{1}{3}(x - 7)$

$y + 4 = -\frac{1}{3}(x - 7)$

$3(y + 4) = -x + 7$

$3y + 12 = -x + 7$

$3y + x = -5$

concurrent - 3 lines intersect at same point

AD : $2y - x = -5$ ①

BE : $y = -2x$

$y + 2x = 0$ ②

$4y - 2x = -10$ ③ (① × 2)

$5y = -10$ ② + ③

$y = -2$

sub $y = -2$ in ②

$-2 + 2x = 0$

$2x = 2$

$x = 1$

∴ $(1, -2)$ is point of intersection of AD & BE.

13F

6 continued (iv)

check $(1,-2)$ lies on CF

$3y+x$

$=3(-2)+1$

$=-5$ ✓

so ~~3y~~ ~~x,~~ $(1,-2)$ lies on $3y+x=-5$ and the centroid coordinates will be $\underline{(1,-2)}$.